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6. AUTHOR(S)
Andrew F. Bennett

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Oregon State University
Corvallis, OR 97331

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Regularly-spaced Eulerian arrays of current meters are becoming rare, while logarithmic arrays are all but unknown. A global subsurface ocean observing system is only a vision at this time. Subsurface floats are increasingly being deployed, albeit sparingly, in many ocean current systems. It is therefore imperative to be able to make dynamically sound estimates of current shears and pressure gradients from float data. It is obvious that a pair of floats provides Lagrangian spatial gradients, for any parameter measured by both floats, but only in the direction of initial separation. A cluster of three or more floats would yield the gradient in all directions.
The scientific objective is the estimation of all components of Lagrangian spatial gradients using a float pair, by exploiting the Lagrangian formulation of fluid dynamics. The technical objective is the computation of these gradients using low-order dynamics, in a high-level programming environment on a Personal Computer.

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Ocean Dynamics from Float Pairs

Andrew F. Bennett
College of Oceanic & Atmospheric Sciences
104 Ocean Admin Bldg
Corvallis OR 97331-5503
phone: (541) 737-9352 fax: (541) 737-2064 email: bennett@coas.oregonstate.edu

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LONG-TERM GOALS

Ocean dynamics can only be understood by measuring current shear and pressure gradient. The long-term goal of this investigation is the economical measurement and efficient dynamical estimation of both.

OBJECTIVES

Regularly-spaced Eulerian arrays of current meters are becoming rare, while logarithmic arrays are all but unknown. A global subsurface ocean observing system is only a vision at this time. Subsurface floats are increasingly being deployed, albeit sparingly, in many ocean current systems. It is therefore imperative to be able to make dynamically sound estimates of current shears and pressure gradients from float data. It is obvious that a pair of floats provides Lagrangian spatial gradients, for any parameter measured by both floats, but only in the direction of initial separation. A cluster of three or more floats would yield the gradient in all directions.

The scientific objective is the estimation of all components of Lagrangian spatial gradients using a float pair, by exploiting the Lagrangian formulation of fluid dynamics. The technical objective is the computation of these gradients using low-order dynamics, in a high-level programming environment on a Personal Computer.

APPROACH

The approach has two parts: first the Lagragian formulation, and second the optimization problem.

The formulation employs the Cauchy-Weber integrals in a rotating reference frame on the sphere, the conservation of volume and the conservation of the Cauchy vector (Lamb, 1932; Batchelor, 1973; Bennett, 2006). Together with the observations from the float pair, these constraints all yield either algebraic equations, or ordinary differential equations with respect to time, for the Jacobi matrix elements and the Lagrangian pressure gradient. The system is rank-deficient, and a weighted least-squares solution is sought. The fitting problem is small but complicated. Direct minimization is both feasible

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and preferable, and within the capability of a PC.

WORK COMPLETED

Year 1. The formulation has been derived on the beta-plane for both shallow-water dynamics, and for fully-stratified primitive-equation dynamics. A shallow-water formulation on the sphere has been coded in MATLAB, and is being run on a 2 GHz Pentium-4 PC under Windows XP. Tests are under way with data from the AMUSEAC float data from the North Atlantic Current Experiment (Rossby, 1996). Surprisingly, the identity matrix is not the Jacobi matrix of smallest norm consistent with the linear constraints at launch time. The null space must therefore be suppressed; this may be achieved either formally, or by progressive introduction of the nonlinear constraints (conservation of volume, and conservation of the Cauchy vector). Quasi-Newton methods for minimization are being tested.

Year 2. Introduction of the law of conservation of the vertical component of the Cauchy vector converts the mathematical problem, from a synoptic analysis of a number of time series, to fixed interval smoothing of a number of series. The optimization problem is very substantially more complex, although quite feasible on a small computer. Accordingly a *poor man's float pair analysis* has been developed. The assumption of quasigeostrophy leads to a simple synoptic relation between the Cauchy invariant and the layer thickness. The geostrophic momentum equations are accordingly simplified, and added as further linear constraints. They are not, in light of the already-imposed Cauchy-Weber first integrals, redundant. The unknowns here are not time dependent fields over labeling space, but are a finite number of time series. Indeed, the imposition of geostrophic momentum is equivalent to imposing the product rule for label differentiation upon the time series. {To explain, it is equivalent to constructing the time series $d(xy)/da$ from the time series for x, dx/da, y, and dy/da, by using $d(xy)/da=(dx/da)y+x(dy/da)$.} There are then 7 linear constraints, of rank 5, for 6 unknowns. The two left null vectors, and the single right null vector, have been identified in the general case. The generalized inverse for the float pair problem (that is, the Jacobi matrix and the gradient of the Cauchy-Weber integral scalar), is then uniquely determined by choosing the projection on the right null vector so that conservation of volume is exactly satisfied, as is feasible. The float path corresponding to the right null vector is a singular transformation of the release coordinates, and so the conservation of volume constraint on the right null vector is linear rather than quadratic. The analysis is purely synoptic; the Lagrangian gradients may be determined at any time along the float paths.

Year 3. The search for the multiple of the right null vector that yields exact conservation of volume is very poorly conditioned, owing to the tendency of the solution to rotate towards the right null vector. An alternative determination of the solution has been devised: the quadratic constraint imposed by conservation of volume upon the Jacobi matrix may be resolved into two alternative linear constraints, both of which are imposed. Best fits to the 9 linear constraints are found with the generalized matrix inverse. There are three left null vectors and no right null vectors, that is, the best-fit is unique. Weighting the fit is based on dynamical scaling.

RESULTS

Results for the NAC data indicate that it is indeed possible to reconcile observations of pairs of deep isopycal floats with Lagrangian shallow-water dynamics, and simultaneously to estimate the both the entire Jacobi matrix and the entire Lagrangian pressure gradient along the float path centroid. A lengthy paper (Bennett, 2007) is in an advanced state of preparation for submission for publication; draft copies are available from the author.

IMPACT/APPLICATIONS

A new combination of classical but long-ignored fluid dynamics is used, together with archived high-quality float-pair data and a powerful direct optimization method, to estimate ocean dynamics in the neighborhood of float pairs. A low-noise, code-simple and cheap tactical system for ocean estimation has been produced.

The theory was presented at the International Summer School on Lagrangian Oceanography, conducted from July 25 to August 5 of 2005 at the Graduate School of Oceanography, by Professor H. Thomas Rossby and myself. The summer school was funded principally by ONR and to a lesser extent by the NOAA Sea Grant to Oregon State University.

RELATED PROJECTS

A related activity has been the writing of a research monograph on Lagrangian fluid dynamics (Bennett, 2006). The last chapter introduces the theory applied here.

PUBLICATIONS

Bennett, A. F., 2007: Lagrangian Analysis of Float Pairs. (*In preparation*).

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